Additional comments of the Logic of Comparative Theory Evaluation

1.) Reevaluation of Schaffner

2.) Two interpretations of ε
(a) Urn model → e is only randomly selected prediction of T
(b) $P(e/T_i) = 0$ or 1 → e is a given prediction of T.

→ Analysis of rag bag of unformulated theories

3.) $\varepsilon = 1$ is a necessary condition for adhoc
Is it a sufficient condition?
i.e. does non-adhoc → $\varepsilon \neq 1$
This would be the case if we take as application of ~~non-ad~~ adhoc $\varepsilon = 1$ for all of a large class of individual investigators)
i.e. if $\varepsilon = 1$ is intersubjectively const.
This is application of principle of sufficient reason (effect → cause ≡ no cause → no effect
Here (no alternative e in background knowledge) → no consensus not to consider alternative theories which do not explain e)

4.) Note on quantitative prediction

$$\varepsilon^n \rightarrow \left(\frac{1}{n}\right)^{n_1} = \frac{1}{n^{n_1}}$$

(or better $\frac{1}{n-1} \cdot \frac{1}{n^{n_1-1}}$

but n^{n_1} is least upper bound for N ensembles in form n^n. If we identify [illegible] each e.v.? with N
we have $\frac{1}{n^{n_1}} = \text{const}$.
(assuming 1st digit not equal to zero.

Reevaluation of Schaffner

Two versions *strong* and *weak*

Strong version was a rational reconstruction of Schaffner's argument thus:–

Schaffner wanted to explicate ad hoc$_2$.

Since he used a wrong formulation of Bayes' theorem involving $P(e) = \varepsilon(1-x)$ he could not explicate ad hoc$_2$ as $x=1$ without making $\varepsilon \to 1$ and $x \to 0$. Hence his requirement for ad hocness ~~[illegible]~~ involved low values of x, but this was confusing ad hoc$_3$ with ad hoc$_2$.

Weak version Schaffner is very ambiguous (difficulties of understanding him are not philosophical or mathematical, but in a sense grammatical)

By *other theories* in his formulation of Bayes' theorem he means other background theories, *not* the alternatives under evaluation.

In spite of promising notion of ad hoc$_2$ his formal analysis of ad hoc (omitting suffix) is not intended to apply to ad hoc$_2$. For Schaffner a theory (or hypothesis) is ad hoc if has a low value of x and $P(T/e)$ remains low.

ie. he requires $\frac{x}{p(e)} = \lambda x$ to be small.

He assumes $p(e) \gg x$ because $p(e|T_i)$ for some alternatives i actually considered is large.

Schaffner is thus concerned with absolute degree of confirmation, not the enhancement ratio.

Our main comment here is that the analysis of adhoc$_2$ does not depend on x being small and applies to any theory in relation to evidence e.

* we could reformulate theory in terms of 2-outcome "events"
i.e. instead of asking what value does a parameter have we can ask does the parameter lie in such, such a range.
→ yes-no propositions linked by connectives & partially ordered by implications → Boolean lattice of propositions of Birkhoff & von Neumann

Two interpretations of E

Consider a simple model for ~~solo~~ the situation we are discussing

Suppose there are M possible events
(e.g. we can think of a set of parameters predicted quantitatively by the theory such as a set of wavelengths of spectral lines)

Suppose for each event there are N possible results *
(e.g. we might ~~predict~~ to measure to 1 place of decimals so that we get 10 possibilities allowed for our accuracy of measurement)

Total no of theories is number of functions from {M} into {N}

$$\text{ie. } \overline{\{f : \{M\} \to \{N\}\}} = N^M$$

(Actually theories correspond to sets of functions but we ignore this complication for the moment)

~~No of simple theories (represented as straight lines on the diagram) is N^2~~

~~Each simple theory gives one prediction for each event so no of "points" for N^2 lines is N^3~~

~~Hence approximate no of lines "near" each point is~~

~~No of true predictions is~~

‡ note $\varepsilon \simeq \frac{1}{N}\left(1 - \frac{1}{N^{M-1}}\right) < 1/N$

this is because there are now fewer false theories giving a true prediction than false theories giving a false prediction

so Prob(true prediction / false theory) is depressed below $1/N$

* note that each particular theory that predicts result e gets enhanced by same factor

So for theory being considered

$$p(T/e) = \lambda x$$

for $(m-1)$ successful alternative theories

$$p(T_i/e) = \lambda \cdot \frac{1-x}{n-1}$$

and we require $p(T/e) + \sum_{i\neq 1} P(T_i/m) = 1$

i.e. $\lambda\left(x + \frac{m-1}{n-1}\cdot(1-x)\right) = 1$

or $\lambda = \dfrac{1}{x + \frac{m-1}{n-1}(1-x)}$ as we expect.

~~Number of theories through a given point in M N plane is~~

$$N^{M-1}$$

~~∴ ratio~~ $\cfrac{\text{no. of false theories through a "true" point}}{\text{no. of false theories}}$

~~is~~

$$\frac{N^{M-1}-1}{N^{M}-1} \simeq \frac{1}{N}$$

~~∴ probability that a particular false theory will make a particular true prediction is $\frac{1}{N}$ if N and M are large.~~
~~(e.g. $N \geq 2$, M large)~~

$P(e|T_i) = 0$ or 1

Indeed suppose there are m false theories which make a particular true prediction e and there are n false theories in all.

So $m = N^{M-1}$ etc in our example
$n = N^{M}$ etc

Give equal weight to each of $(n-1)$ ~~false~~ theories and weight x to theory under consideration

So $P(T_1|h) = x$, $P(T_i|h) = \frac{1-x}{n-1}$ $i \neq 1$

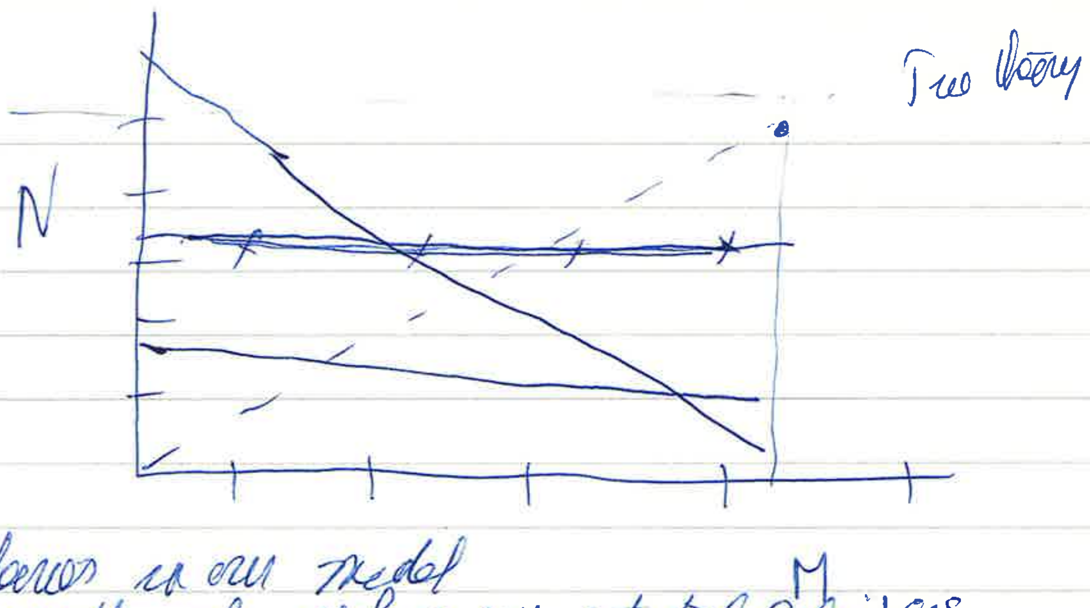
Then $p(e|h) = P(e|T_1 \& h)P(T_1|h) + \sum_{i \neq 1} P(e|T_i \& h)P(T_i|h)$

$$= x + \frac{1-x}{n-1} \cdot m-1$$

$$= x + \left(\frac{m-1}{n-1}\right)(1-x) = x + \left(\frac{N^{M-1}-1}{N^{M}-1}\right)(1-x) \qquad \text{II}$$

$$\simeq x + \varepsilon(1-x) \quad \text{with } \varepsilon = 1/N.$$

*



Note all theories in our model are equally simple – they all have NM potential falsifiers.

We can look at interpretation of E in another way

New model

In previous model e is a particular prediction of theory T_1.

We now suppose e is any randomly selected true prediction of T_1. We now consider another theory T_i and ask what is probability of e agreeing with the prediction of T_i for the same event, where i has any value different from 1.

no of predictions of T_1 is M

no of (predictions of T_1, alternative theory) pairs is $(n-1)M$

no of (predictions of T_1, prediction of T_i $i \neq 1$) pairs is $M.(n-1)M = (n-1)M^2$

no of (predictions of T_1, prediction of T_i agreeing with that of T_1) pairs $= M(m-1)$.

∴ probability of prediction of alternative theory agreeing with prediction of given theory $= \frac{\cancel{n-1}(m-1)M}{(n-1)M}$

$= \frac{m-1}{n-1} \approx \frac{1}{N}$ as before.

this situation can be interpreted as follows.

~~Total number of urns = N~~ Ω

~~x Ω urns are r~~

Each theory is represented by an urn of different (unknown) colours.
Each urn is filled with balls of different (unknown) colours.

the "true" urn is coloured red and filled with red balls.
the remaining urns have fraction ε of red balls in them, a red ball corresponding to a true prediction'.

We are asking what is chance if we draw a red ball that it came' from a red urn.

~~Note prob. of given alternative theory to make a prediction agreeing with the prediction of T_1 is $\frac{1}{N}$~~

also $P(e/T_2 \vee T_3 \ldots \vee T_n) = \dfrac{P(e \wedge (T_2 \vee T_2 \ldots \vee T_n))}{P(T_2 \vee T_2 \ldots T_n)}$

$$= \frac{P((e \wedge T_2) \vee (e \wedge T_3) \ldots)}{P(T_1 \vee T_2 \ldots)} = P(e/T_2)\,\frac{P(T_2)}{P(T_2)+P(T_3)+\ldots} \nearrow \text{small factor}$$

$$+ P(e/T_3)\,\frac{P(T_3)}{P(T_2)+P(T_3)} \ldots$$

if $P(e/T_i)_{i \neq 1} = \varepsilon$, ~~ad~~

then clearly $P(e/T_2 \vee T_3 \vee \ldots T_n) = P(e/T_i)_{i \neq 1} = \varepsilon$.